ON THE STABILIZATION OF A NONLINEAR CONTROLLED SYSTEM IN THE CRITICAL CASE OF ZERO AND PURELY IMAGINARY ROOTS

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The problem of stabilization of steady motions of a nonlinear controlled system in the critical case of h zero and k pairs of purely imaginary roots ($h > 0$, $k > 0$ are integers) is considered.

A continuous control is introduced. The control in question is analytic in the h variables corresponding to the zero roots and nonanalytic in the $2k$ variables corresponding to the imaginary roots of the characteristic equation of the linear part of the system. The analysis is based on the classical theory of Liapunov stability of motion [1] and on the methods developed in [2].

1. Let us consider the controlled system

$$dw/dt = Av + Bu + g(v, u) \tag{1.1}$$

Here w is the $(n + h + 2k)$-dimensional perturbation vector; u is the m-dimensional control vector, which we assume to be unaffected by interference; A, B are constant matrices of the appropriate dimensions. We assume that all the coefficents of Eq. (1.1) are real and that $g(w, u)$ is a vector function analytic in v and u whose expansion in powers of w, u begins in terms of order not lower than the second.

If the unperturbed motion $w = 0$ of system (1.1) is not asymptotically stable for $u \equiv 0$, then there arises the problem of stabilization, i.e. the problem of choosing a control $u = u(w)$ whose substitution into (1.1) makes the zero solution $w = 0$ asymptotically stable in the Liapunov sense.

Let us consider the critical case of h zero and k pairs of purely imaginary roots [2]. We assume that the h zero roots correspond to h groups of solutions. In this case a suitable choice of variables allows us to rewrite system (1.1) in the form

$$dz_j/dt = Z_j(x_i, y_i, z_j, v, u), \qquad dx_i/dt = -\lambda_i y_i + X_i(x_i, y_i, z_j, v, u)$$

$$dy_i/dt = \lambda_i x_i + Y_i(x_i, y_i, z_j, v, u) \tag{1.2}$$

$$dv/dt = A_0 v + B_0 u + \Sigma(a_i x_i + b_i y_i) + \Sigma c_j z_j + \Omega(x_i, y_i, z_j, v, u) \tag{1.3}$$

Here x_i, y_i, z_j are scalar variables; v is an n-dimensional vector with the components v_σ; a_i, b_i, c_j are n-dimensional constant vectors; A_0, B_0 are constant matrices of order $n \times n$ and $n \times m$, respectively; Ω is a vector function with the components Ω_σ; the functions X_i, Y_i, Z_j, Ω_σ are analytic nonlinearities in x_i, y_i, z_j, v, w; λ_i/λ_ν is irrational; the subscript i runs through the values $1,2,\ldots,k$, the subscript j through the values $1,2,\ldots,h$, and the subscript σ through the values $1,2,\ldots,n$.

The stabilization problem for system (1.1) is equivalent to the same problem for system (1.2), (1.3).

As we know [2], the system

$$dv/dt = A_0 v + B_0 u \tag{1.4}$$

is stabilizable and admits of the linear control

$$I^\circ(v) = P_v \qquad (1.5)$$

The constant matrix P of order $m \times n$ must be chosen in such a way that when (1.5) is substituted into (1.4) all the eigenvalues of the matrix $C = A_0 + B_0 P = \text{const}$ ($C = (e_{i\nu})$) have negative real parts.

For system (1.2), (1.3) we use a nonanalytic equation of the form

$$u(x_i, y_i, z_j, v) = Pv + \theta(x_i, y_i, z_j) \qquad (\theta = (\theta_1, \ldots, \theta_m)) \qquad (1.6)$$

$$\theta_\mu(x_i, y_i, z_j) = \theta_\mu^{(1)} + \theta_\mu^{(2)} + \ldots + \theta_\mu^{(\delta_1)} \qquad (1.7)$$

$$\theta_\mu^{(r)}(x_i, y_i, z_j) = \sum_{l=-1}^{a_{\mu r}} \sum_r \alpha_\mu^{(\tau)} x_1^{p_1} y_1^{q_1} z_1^{s_1} \ldots x_k^{p_k} y_k^{q_k} z_h^{s_h} \rho^{-l} \qquad (1.8)$$

$$\sum(p_i + q_i) + \sum s_j - l = r, \qquad (\tau) = (p_1 q_1 s_1 \ldots p_k q_k s_h - l)$$

$$p_i \geqslant 0, \quad q_i \geqslant 0, \quad s_j \geqslant 0, \quad \delta_1 > 0, \quad a_{\mu r} = \text{const} \geqslant 0 - \text{ are integers}$$

$$\rho = [\Sigma \beta_i (x_i^2 + y_i^2)]^{1/2}, \qquad \beta_i = \text{const} > 0$$

$$(r = 1, 2, \ldots, \delta_1; \quad \mu = 1, 2, \ldots, m)$$

The following characteristic estimates for homogeneous rth order forms are valid for functions of the form (1.8):

$$|\theta_\mu^{(r)}(x_i, y_i, z_j)| \leqslant A_\mu^r \|\chi\|^r; \quad \|\chi\| = \sqrt{\Sigma(x_i^2 + y_i^2) + \Sigma z_j^2}; \quad A_\mu^r = \text{const} > 0$$

Here and below we assume that

$$\theta(0, 0, 0) = \lim \theta(x_i, y_i, 0) = 0 \qquad (x_i \to 0, \; y_i \to 0)$$

The constants $\alpha_\mu^{(\tau)}$ and the integers δ_1, $a_{\mu r}$ will be chosen in accordance with the form of system (1.2), (1.3) and the possibility of its stabilization.

Let us transform system (1.2), (1.3) in such a way that the equations for the noncritical variables in the transformed system do not contain terms of order lower than N ($N > 1$ is an integer) which depend only on x_i, y_i, z_j. This can be done with the aid of the transformation

$$v_\sigma = \xi_\sigma + \varkappa_\sigma(x_i, y_i, z_j) \qquad (1.9)$$

where ξ_σ are the new variables. To determine the functions $\varkappa_\sigma(x_i, y_i, z_j)$ in accordance with the Liapunov method we consider the system of partial differential equations

$$\Sigma \frac{\partial \varkappa}{\partial z_j} Z_j(x_i, y_i, z_j, \varkappa, u) + \Sigma \frac{\partial \varkappa}{\partial x_i}[-\lambda_i y_i + X_i(x_i, y_i, z_j \varkappa, u)] +$$

$$+ \Sigma \frac{\partial \varkappa}{\partial y_i}[\lambda_i x_i + Y_i(x_i, y_i, z_j, \varkappa, u)] = A_0 \varkappa + B_0 u + \Sigma(a_i x_i + b_i y_i) +$$

$$+ \Sigma c_j z_j + \Omega(x_i, y_i, z_j, \varkappa, u) \qquad (1.10)$$

where $\varkappa$ is an n-dimensional vector with the components $\varkappa_\sigma$. We shall seek the solution of this system in the form of the formal series

$$\varkappa_\sigma(x_i, y_i, z_j) = \varkappa_\sigma^{(1)} + \varkappa_\sigma^{(2)} + \ldots \qquad (1.11)$$

where $\varkappa_\sigma^{(r)}$ are functions of the type (1.8), i.e.

$$\varkappa_\sigma^{(r)}(x_i, y_i, z_j) = \sum_{l=-1}^{b_{\sigma r}} \sum_r a_\sigma^{(\tau)} x_1^{p_1} y_1^{q_1} z_1^{s_1} \ldots x_k^{p_k} y_k^{q_k} z_h^{s_h} \rho^{-l} \qquad (1.12)$$

$$(p_1 + q_1 + s_1 + \ldots + p_k + q_k + s_h - l = r; \; (\tau) = (p_1 q_1 s_1 \ldots p_k q_k s_h - l))$$

Substituting these series and control (1.6), (1.8) into (1.10), and then equating the νth order terms (for which $p_1 + q_1 + s_1 \ldots + p_k + q_k + s_h - l = \nu$) in the left and right sides of the resulting equations, we obtain the following system for determining the vector function $\varkappa^{(\nu)}$:

$$\Sigma\lambda_i\left(x_i \frac{\partial\varkappa^{(\nu)}}{\partial y_i} - y_i \frac{\partial\varkappa^{(\nu)}}{\partial x_i}\right) = C\varkappa^{(\nu)} + \tau^{(\nu)}(x_i, y_i, z_j, \varkappa^{(\gamma)}) \qquad (1.13)$$

The components $\tau_\sigma^{(\nu)}$ are vector functions, $\tau^{(\nu)}$ are νth order homogeneous functions of the variables x_i, y_i, z_j of the (1.8) type. For example, for $\nu = 1$ we have

$$\tau^{(1)} = \Sigma\,(a_i x_i + b_i y_i) + \Sigma\, c_j z_j + B_0\theta^{(1)}$$

The functions $\tau^{(\nu)}$ for $\nu > 1$ depend only on those $\varkappa_\sigma^{(\gamma)}$ for which $\gamma < \nu$. Since we assume that all the functions $\varkappa_\sigma^{(\gamma)}$ for $\gamma < \nu$ have already been computed, it follows that the functions $\tau_\sigma^{(\nu)}$ are known.

Isolating the terms with equal factors ρ^{-l} in the functions $\varkappa_\sigma^{(\nu)}$ and $\tau_\sigma^{(\nu)}$, we can express them in the form

$$\varkappa_\sigma^{(\nu)} = \sum_{l=-1}^{b_{\sigma\nu}} \varkappa_\sigma^{(\nu+l)_\nu}\rho^{-l}, \qquad \tau_\sigma^{(\nu)} = \sum_{l=-1}^{\pi_{\sigma\nu}} \tau_\sigma^{(\nu+l)_\nu}\rho^{-} \qquad (1.14)$$

Here $\pi_{\sigma\nu} \geqslant 0$ are integers; $\varkappa_\sigma^{(\nu+l)_\nu}$ and $\tau_\sigma^{(\nu+l)_\nu}$ are $(\nu + l)$-th order forms in x_i, y_i, z_j. Substituting (1.14) into (1.13) and recalling that

$$\Sigma\lambda_i\left(x_i \frac{\partial\rho^{-l}}{\partial y_i} - y_i \frac{\partial\rho^{-l}}{\partial x_i}\right) \equiv 0$$

we obtain

$$\sum_{l=-1}^{b_{\sigma\nu}} \sum_{i=1}^{k} \lambda_i\left(x_i \frac{\partial\varkappa_\sigma^{(\nu+l)_\nu}}{\partial y_i} - y_i \frac{\partial\varkappa_\sigma^{(\nu+l)_\nu}}{\partial x_i}\right)\rho^{-l} = \qquad (1.15)$$

$$= \sum_{l=-1}^{b_{\sigma\nu}} \sum_{s=1}^{n} c_{\sigma s}\varkappa_\sigma^{(\nu+l)_\nu}\rho^{-l} + \sum_{l=-1}^{\pi_{\sigma\nu}} \tau_\sigma^{(\nu+l)_\nu}\rho^{-}$$

To determine the numbers $b_{\sigma\nu}$ we first set

$$b_{1\nu} = b_{2\nu} = \ldots = b_{n\nu} = \max\,[\pi_{1\nu}, \pi_{2\nu}, \ldots, \pi_{n\nu}]$$

in (1.12), (1.15) and then obtain specific values for these constants by equating the terms with equal factors ρ^{-l} in the right and left sides of Eqs. (1.15). This gives us the following equations for determining the vector functions $\varkappa^{(\nu+l)_\nu}$:

$$\Sigma\lambda_i\left(x_i \frac{\partial\varkappa^{(\nu+l)_\nu}}{\partial y_i} - y_i \frac{\partial\varkappa^{(\nu+l)_\nu}}{\partial x_i}\right) = C\varkappa^{(\nu+l)_\nu} + \tau^{(\nu+l)_\nu} \qquad (1.16)$$

This system is particular case (32) of [1], Sect. 30 (see also (39.1) of [3], Sect. 39). By virtue of the Liapunov theorem [1, 3] system (1.16) has a unique solution for the forms $\varkappa_\sigma^{(\nu+l)\nu}$. This solution can be obtained by the method of undetermined coefficients; this yields linear nonhomogeneous algebraic systems for determining the coefficients of the forms in question. Thus, Eqs. (1.16) make it possible to determine successively the forms $\varkappa_\sigma^{(\nu+l)_\nu}$ ($\nu = 1, 2, \ldots$) (and therefore the functions $\varkappa_\sigma^{(r)}$ (1.12)).

At this point we can show that if

$$\rho = \sqrt{\Sigma\,(\alpha_i x_i^2 + \beta_i y_i^2)}$$

is substituted into control (1.6) – (1.8), then it is necessarily the case that $\alpha_i = \beta_i$.

Let us suppose that all the functions $\varkappa_\sigma\,(x_i, y_i, z_j)$ of up to a prescribed order $N - 1$ have already been computed, i.e. that

$$\varkappa_\sigma\,(x_i, y_i, z_j) = \varkappa_\sigma^{(1)} + \varkappa_\sigma^{(2)} + \ldots + \varkappa_\sigma^{(N-1)} \qquad (1.17)$$

are known.

Substituting control (1.6)–(1.8) into Eqs. (1.2), (1.3) and transforming this system in accordance with formulas (1.9), (1.17), we obtain

$$\frac{dz_\beta}{dt} = \sum_{r=2}^{N} R_\beta^{(r)}(x_i, y_i, z_j) + \varphi_\beta(x_i, y_i, z_j, \xi)$$

$$\frac{dx_\alpha}{dt} = -\lambda_\alpha y_\alpha + \sum_{r=2}^{N} H_\alpha^{(r)}(x_i, y_i, z_j) + \psi_{1\alpha}(x_i, y_i, z_j, \xi) \qquad (1.18)$$

$$\frac{dy_\alpha}{dt} = \lambda_\alpha x_\alpha + \sum_{r=2}^{N} K_\alpha^{(r)}(x_i, y_i, z_j) + \psi_{2\alpha}(x_i, y_i, z_j, \xi)$$

$$\frac{d\xi}{dt} = C\xi + \Omega^*(x_i, y_i, z_j, \xi) \qquad (\Omega^* = (\Omega_1^*, \ldots, \Omega_n^*))$$

$$(\beta = 1, 2, \ldots, h; \quad \alpha = 1, 2, \ldots k)$$

Here the functions $\varphi_\beta, \psi_{1\alpha}, \psi_{2\alpha}, \Omega_\sigma^*$ are of an order of smallness not lower than the second in $x_i, y_i, z_j, \xi_\sigma$.

The functions $\varphi_\beta(x_i, y_i, z_j, 0)$, $\psi_{1\alpha}(x_i, y_i, z_j, 0)$, $\psi_{2\alpha}(x_i, y_i, z_j, 0)$ satisfy the Lipschitz condition with an infinitely small constant and the estimates

$$|\varphi_\beta(x_i, y_i, z_j, 0)| \leqslant A_\beta \|\chi\|^{N+1}, \qquad |\psi_{1\alpha}(x_i, y_i, z_j, 0)| \leqslant B_{1\alpha}\|\chi\|^{N+1}$$

$$|\psi_{2\alpha}(x_i, y_i, z_j, 0)| \leqslant B_{2\alpha}\|\chi\|^{N+1}, \qquad A_\beta > 0, \quad B_{1\alpha} > 0, \quad B_{2\alpha} > 0 \text{ (const)}$$

As a result of our choosing transformation (1.9), (1.17), the expansion of the function $\Omega_\sigma^*(x_i, y_i, z_j, 0)$ begins with terms of order not lower than N.

Fulfillment of these conditions ensures the applicability of Theorem 2.2 of [4]. In other words, the problem of stability of the zero solution of system (1.18) is equivalent to the problem of stability of the zero solution of the truncated system

$$\frac{dz_\beta}{dt} = \sum_{r=2}^{N} R_\beta^{(r)}(x_i, y_i, z_j) \qquad \frac{dx_\alpha}{dt} = -\lambda_\alpha y_\alpha + \sum_{r=2}^{N} H_\alpha^{(r)}(x_i, y_i, z_j)$$

$$\frac{dy_\alpha}{dt} = \lambda_\alpha x_\alpha + \sum_{r=2}^{N} K_\alpha^{(r)}(x_i, y_i, z_j) \qquad (1.19)$$

We can obtain system (1.19) from system (1.2) by setting control (1.6) – (1.8) into the latter, replacing the components of the vector z in the resulting relations by the components of the vector $\varkappa$ (1.17), and retaining terms of order up to N only.

2. Let us consider the stability of truncated system (1.19). We can rewrite the system as

$$\frac{dz_j}{dt} = R_j^{(2)}(x_i, y_i, z_i) + R_j^{(3)}(x_i, y_i, z_j) + \ldots$$

$$\frac{dx_i}{dt} = -\lambda_i y_i + H_i^{(2)}(x_i, y_i, z_j) + H_i^{(3)}(x_i, y_i, z_j) + \ldots \qquad (2.1)$$

$$\frac{dy_i}{dt} = \lambda_i x_i + K_i^{(2)}(x_i, y_i, z_j) + K_i^{(3)}(x_i, y_i, z_j) + \ldots$$

where $R_j^{(r)}, H_i^{(r)}, K_i^{(r)}$ is the set of rth order terms of the (1.8) type whose coefficients depend in a certain way on the coefficients of control (1.6) – (1.8).

Kamenkov [5] investigated system (2.1) in the class of analytic functions by a method requiring a large number of preliminary transformations.

We shall confine our attention to the case where the possibility of stabilizing system (2.1) is determined by the second-order terms $R_j^{(2)}$, $H_i^{(2)}$ $K_i^{(2)}$. In this case control (1.6) - (1.8) need contain first-order terms only. Moreover, if we limit ourselves to the values $l = -1, 0$, we need only consider the control

$$u_\mu = u_\mu^0 (v) + \sum_1 \alpha_\mu^{(\tau_1)} x_1^{p_1} y_1^{q_1} z_1^{s_1} \ldots x_k^{p_k} y_k^{q_k} z_h^{s_h} + \alpha_\mu^{(\tau_2)} \rho$$

$$(p_1 + q_1 + s_1 + \ldots + p_k + q_k + s_h = 1; \quad (\tau_1) = (p_1 q_1 s_1 \ldots p_k q_k s_h 0)) \tag{2.2}$$

$$(\tau_2) = (0\,0\,0 \ldots 0\,0\,0\,1)$$

Such a choice of control (2.2) yields

$$R_j^{(2)} = \tau_j^{(1)} (x_i, y_i, z_j) \rho + \tau_j^{(2)} (x_i, y_i, z_j), \qquad H_i^{(2)} = \varphi_i^{(1)} (x_i, y_i, z_j) \rho + \varphi_i^{(2)} (x_i, y_i, z_j) \tag{2.3}$$

$$K_i^{(2)} = \psi_i^{(1)} (x_i, y_i, z_j) \rho + \psi_i^{(2)} (x_i, y_i, z_j)$$

Here $\tau_j^{(\delta)}$, $\varphi_i^{(\delta)}$, $\psi_i^{(\delta)}$ $(\delta = 1, 2)$ are δth order forms in x_i, y_i, z_j. The coefficients of these forms depend in a certain way on the coefficients of control (2.2).

Let us consider the Liapunov function of the form

$$2V = \Sigma (x_i^2 + y_i^2) + \Sigma z_j^2 + 2W (x_i, y_i, z_j)$$

where W is a third-order form in x_i, y_i, z_j. We shall attempt to choose this form in such a way that the total derivative of the function V is of fixed sign by virtue of Eqs. (2.1). This derivative can be written as

$$\frac{dV}{dt} = \Sigma z_j R_j^{(2)} + \Sigma (x_i H_i^{(2)} + y_i K_i^{(2)}) + \Sigma \lambda_i \left(x_i \frac{\partial W}{\partial y_i} - y_i \frac{\partial W}{\partial x_i} \right) + \ldots$$

where the ellipsis represents terms of order higher than the third.

Making use of expressions (2.3), we can write

$$\frac{dV}{dt} = \varphi (x_i, y_i, z_j) \rho + \Phi (x_i, y_i, z_j) + \Sigma \lambda_i \left(x_i \frac{\partial W}{\partial y_i} - y_i \frac{\partial W}{\partial x_i} \right) + \ldots \tag{2.4}$$

Here φ is a quadratic form and Φ is a third-order form in x_i, y_i, z_j. The coefficients of the form W can be chosen in such a way as to satisfy the equation

$$\Sigma \lambda_i \left(x_i \frac{\partial W}{\partial y_i} - y_i \frac{\partial W}{\partial x_i} \right) + \Phi (x_i, y_i, z_j) = \sum_{\alpha, \beta, \gamma = 1}^{h} a_{\alpha\beta\gamma} z_\alpha z_\beta z_\gamma \tag{2.5}$$

$$(a_{iji} = a_{iij} = a_{jii}; \quad i, j = 1, 2, \ldots, h)$$

In determining $W (x_i, y_i, z_j)$ it is sufficient to isolate from the set of third-order terms of $\Phi (x_i, y_i, z_j)$ (2.4) those terms which depend only on the critical variables z_j written out in the right side of Eqs. (2.5).

The derivative dV / dt now becomes

$$\frac{dV}{dt} = \varphi (x_i, y_i, z_j) \rho + \sum_{\alpha, \rho, \gamma = 1}^{h} a_{\alpha\rho\gamma} z_\alpha z_\beta z_\gamma + \ldots \tag{2.6}$$

where the ellipsis represents terms of order higher than the third.

The quadratic form $\varphi (x_i, y_i, z_j)$ can be written as

$$\varphi(x_i, y_i, z_j) = \sum_{\alpha,\beta=1}^{2k+h} d_{\alpha\beta}\eta_\alpha\eta_\beta \tag{2.7}$$

Here

$$\eta_{2i-1} = x_i, \qquad \eta_{2i} = y_i, \qquad \eta_{2k+j} = z_j$$

We denote the principal minors of its discriminant by

$$\Delta_{\nu\nu} = (d_{ij}) \qquad (d_{ij} = d_{ji}) \qquad (i, j = 1, 2, \ldots, \nu;\ \nu = 1, 2, \ldots, 2k+h)$$

By the Sylvester criterion, form (2.7) is positive-definite if and only if

$$\Delta_{\nu\nu} > 0 \qquad (\nu = 1, 2, \ldots, 2k+h) \tag{2.8}$$

and negative-definite if and only if

$$\Delta_{2p-1,\,2p-1} < 0, \quad \Delta_{2p,2p} > 0 \qquad (p = 1, 2, \ldots, k+h_1) \tag{2.9}$$

$$(h_1 = {}^1/_2 h \text{ for } h = 2l;\ h_1 = {}^1/_2 (h+1) \text{ for } h = 2l+1)$$

We now require that the coefficients $a_{\alpha\beta\gamma}$ (2.6) satisfy the conditions

$$|a_{\alpha\beta\gamma}| < \varepsilon \quad (\varepsilon > 0 \text{ is sufficiently small}) \tag{2.10}$$

According to Lemma 3 of [3], Sect. 7, fulfillment of condition (2.8) or (2.9) and (2.10) implies that dV/dt (2.6) is of fixed sign.

The function V is positive-definite in a sufficiently small neighborhood of $x_i = 0$, $y_i = 0$, $z_j = 0$. The Liapunov theorem on asymptotic stability and the first theorem on instability imply the following: if inequalities (2.9) and (2.10) are fulfilled, then the unperturbed motion of system (2.1) is asymptotically stable; if inequalities (2.8) and (2.10) are fulfilled, then the unperturbed motion is unstable. By virtue of the reduction principle (Theorem 2.2 of [4]), this means that the same statement is valid for the unperturbed motion of system (1.18) and thereby for initial system (1.2), (1.3).

The following theorem summarizes the above results.

Theorem 2.1 (1). Stabilization of the unperturbed motion of system (2.1), and therefore of the unperturbed motion of system (1.2), (1.3) is ensured by control (2.2) provided the coefficients $\alpha_\mu^{(\tau_1)}$, $\alpha_\mu^{(\tau_2)}$ can be chosen in such a way that inequalities (2.9) and (2.10) are satisfied.

(2). If conditions (2.8) and (2.10) are fulfilled for any choice of coefficients $\alpha_\mu^{(\tau_1)}$, $\alpha_\mu^{(\tau_2)}$, then system (2.1) cannot be stabilized by control (2.2).

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